

Lessons

Organisation of the classroom observations

All three of Berenice's research lessons were video recorded, with two researchers in the classroom. Usually one researcher video recorded and the other took notes, and some of the time one or both circulated and observed the students working.

An a priori analysis of the lesson(s)

Berenice taught three FaSMEd lessons. For all FaSMEd lessons, we decided to use lessons that:

- began with a brief whole class introduction in which the teacher assessed the students' knowledge and understanding of the topic to a greater or lesser extent and told the students what they should do in the lesson;
- adopted so-called 'active learning' approaches, with student working in pairs or small groups, mostly to sort or match sets of small cards. This provides the students with the opportunities to discuss and learn from each other, formatively assessing one another to guide some of their choices;
- expected each group to produce a poster with the card matches stuck down on the poster, and these posters might be shared with other groups;
- included a phase of institutionalisation such as sharing posters or going through the answers together.

To some extent, therefore, the didactical choices were already made for Berenice.

The more subtle choices, such as how and when to intervene in small-group discussions or activity, tend to be made by the teacher and are determined by the individual's preferences and their reaction to the data they gather in real time about their students' understanding.

Cards were our main technology; we did not use ICT, apart from display technologies in some classrooms (Berenice's was one of them) but ICT had no real role in the task design. On the other hand, small card sets were central to the design. The use of small cards by students can provide the teacher with a window into the students' thinking.

In a sense Berenice chose the lessons she wanted to teach by telling us which topic she would like the lessons to address. For all three of the lessons, the topic was one her class was addressing at the time so the lesson fitted into the scheme of work. These lessons were all designed as 'one-off' lessons to be taught after the students have already been working on the topic for a little while.

Although the three lessons followed a similar structure, the actual mathematical task the students were asked to do was different in each case. We briefly analyse the tasks below. It is also perhaps useful to consider, *a priori*, the obstacles inherent in each task as identified by Brousseau (1997): ontogenic obstacles which relate to the readiness of students for the task; didactical obstacles which are teaching and classroom obstacles; and epistemological obstacles, which require the students to adapt their strategies to solve a problem. The first two types of obstacles should be avoided, but the third should be present.

Negative numbers in context

This is a card matching activity, with two sets of cards. The first set has the name of a city and either the temperature of the city or a space for the student to fill in the temperature, showing their calculations. The second set consists of arrows connecting two cities. It is designed to challenge students' difficulties in ordering, comparing, adding, and subtracting positive and negative integers. Particular attention is paid to the use of negative numbers on number lines to explore the structures:

$$\textit{starting temperature} + \textit{change in temperature} = \textit{final temperature}$$

$$\textit{final temperature} - \textit{change in temperature} = \textit{starting temperature}$$

$$\textit{final temperature} - \textit{starting temperature} = \textit{change in temperature}$$

This task was used with a Grade 8 class, who had recently worked on negative numbers and knew the 'rules' related to adding and subtracting negative numbers. However, this task takes them beyond applying the rules.

Sifting through and arranging all the information is complex, and arranging the cards so that everything joins up can be seen as challenging, particularly as there are several correct solutions. To some extent, in our view, this might present a didactical obstacle.

In terms of the classroom teaching, this also had little or no experience of working in groups on card-arranging activities, which could also develop into an ontogenic obstacle. *A priori*, therefore, it was possible that the students would not perform as well as hoped.

Interpreting equations

This is a card matching activity which involves matching a set of equations with written descriptions of real life situations. The equations can be linked to the situations because of the way the variables are defined. Some cards have no matches and some cards are left blank meaning that the students would need to make up their own statements (in two cases) and their own equation (in one case). The matches are not one to one. Once the cards are matched they are pasted onto poster paper. The two main ideas in the lesson are that if $y = 4x$ then y is bigger than x and that it is very important to know what a variable refers to (in this case number of items and cost of items).

This task was used with a Grade 8 class who had recently been introduced to the idea of variables so it was unlikely to present ontogenic obstacles. However, the way the cards are designed should present epistemological obstacles, particularly because firstly the students need to recognise equivalence of equations, such as $y = 2x$ and $x = \frac{1}{2}y$, which means that these cards are grouped together, and secondly because students need to write their own statements and equations on the blank cards.

Berenice decided that she did not want to use the blank cards with her class, so she used a set with all equations and statements filled in; this reduced the challenge of the task considerably. *A priori*, therefore, we might expect them to perform well on the task.

Properties of quadrilaterals

In this lesson, learners are given six sets of cards, each in a different colour. To start the sets of cards are presented in a strip (not cut up), and each strip of cards describes a particular quadrilateral (square, rectangle, parallelogram, rhombus, trapezium and kite) by providing some of the properties of this quadrilateral. The cards also provide information about the measurements of the quadrilateral. Students are asked to work in groups to a) identify the rectangle described by each set of properties b) sketch the quadrilateral and c) choose a minimum number of property cards which fully describe the quadrilateral (there is more than one correct answer for this).

The activity is designed to challenge students because they are used to listing the properties of a shape, when they are given the shape. They are less used to working out what the shape is from a list of properties. Further, usually they list as many properties as they are able, but in this activity (second part) they need to think about how few properties are required to be sure that the shape is fully defined. These challenges can be seen as epistemological obstacles. For this class who were more accustomed to group work and card-based activities by the time this lesson was taught, and who had revised quadrilaterals and their properties in the previous lesson, no ontogenic or didactical obstacles were predicted *a priori*.

Organisation of the lessons

Research lesson 1 (Negative numbers in context)

Lesson 1: 8th March 2015

Activity 1: The students were given the questions shown in Figure 4 (see above) to answer individually. Berenice took in the worksheets and assessed them. She did not give them back, but rather gave them to the researchers. Figure 23 below provides examples of the sorts of comments she began writing on them (but she said she decided to stop writing on them because she was just saying the same sorts of things over and over).

(b) What was the change in temperature? Explain your answer.

* -8° because $+3^{\circ}\text{C} + (-3^{\circ}\text{C}) = -5^{\circ}\text{C}$

How would you explain this without using a calculation?

The temperature was 5°C .
What was the final temperature?

Explain your answer:

$5^{\circ}\text{C} - (-11) = 16$ because
 $5^{\circ}\text{C} - (-11) = 5 + 11 = 16^{\circ}\text{C}$

Why do you think this calculation was written the way it is? (Compare with your choice of story question.)

The temperature was -7°C at midnight.
By the next day, the temperature had risen by 11°C .

(a) Some of these calculations show how to figure out the temperature the next day. Circle any that apply.

$7 - 11$ $11 + (-7)$ $(-11) - (-7)$ $(-7) + 11$

(b) What was the temperature the next day? Explain your answer.

11°C . $-7^{\circ}\text{C} + 11^{\circ}\text{C}$ How would you explain this temperature change to someone else?
Did it increase/decrease?

Figure 1: Pre-lesson assessment with teacher's comments

Lesson 2: 9th March 2015

Activity 2: Berenice briefly discussed the pre-lesson assessment task. She told the class that she had noticed that some people struggled with their explanations. She said that in today's lesson she wanted them to think about how they explain things.

Activity 3: Introduction with whole class

Berenice asked for four volunteers to come to the front of the room. She gave each one a card with the name of a city and the city's temperature, and asked the students to arrange themselves in order from lowest to highest (see Figure 24). She questioned the students about whether they agreed, and they said they did. She then held a short discussion about temperature in contexts. For example she asked them at what temperature water boils and freezes. She reminded them that it had been reported that Cape Town's temperature the previous week had broken a record and asked them what it had been (42° in the city centre).

(Presumably she chose these contexts as she thought it likely that the students would relate to them).



Figure 2: Four volunteers arranged in order of temperature

Berenice displayed the slide shown in Figure 6 (above) and questioned the class about how one could describe the difference in temperature between the two cities (London and Beijing). Five or six students had their hands up and she got responses from three or four. The students argued variously (and at some length, perhaps three or four sentences) for +18° and - 18°. It was finally agreed that it was -18° and the students seemed happy as no more questions were asked. They discussed the Paris and Vancouver example briefly and Berenice pointed out that students should read the arrows carefully as sometimes the name of the city is not next its name on the arrow. She also explained that a number sentence could be written on the arrow and discussed what it would look like.

Activity 4: Pair work

Berenice displayed the slide about how to work together (Figure 9 above) and went through it in detail. She explained to the students that they would work in pairs and each pair would be given card sets and paper. She explained that there is a set of blue cards with city names and a set of arrow cards in yellow and pink – and she said that the pink arrows were for use later in the lesson and that the students should put them to one side. She told them that they should arrange the cards on the paper so that each city and each arrow was used and then to stick them down using prestik.

Berenice and Ingrid handed out the cards and other equipment. The students began to work in their pairs and Berenice began to circulate. After a few minutes she stopped the class and said that she had noticed that some pairs were placing the cards together but not taking turns. She pointed out that the instructions on the slide said that they should take turns.

The students continued to work to arrange the cards. Berenice circulated, talking to the pairs and responding to hands up (three or four times). She reminded the class that all the cards should be placed and that they should first place the cards and then beginning filling in the missing information.

Later Berenice asked if anyone had ‘figured out’ where all the arrows go. Some pairs raised their hands and she said they should start to calculate the temperatures and to fill in the number sentences on the arrows.

When the bell rang for the end of the lesson and Berenice asked the students to write their names on the posters as they would be continuing the lesson the next day.

Lesson 3: 10th March 2015

Activity 4 continued: Berenice went through the activity’s instructions again. Two volunteers handed out the posters to the different groups. With the poster in front of them, some groups changed the arrangement of cards they had made the previous day. As per instruction, learners calculated the different temperatures on the yellow cards. Berenice continued to move from group to group, talking to the learners.



Figure 3: Students working in pairs

Berenice called for attention and asked learners to now use glue to stick their cards, replacing the temporary prestik they had been using. She said they had five minutes to glue their cards onto the posters. Figure 26 shows one of the completed posters.

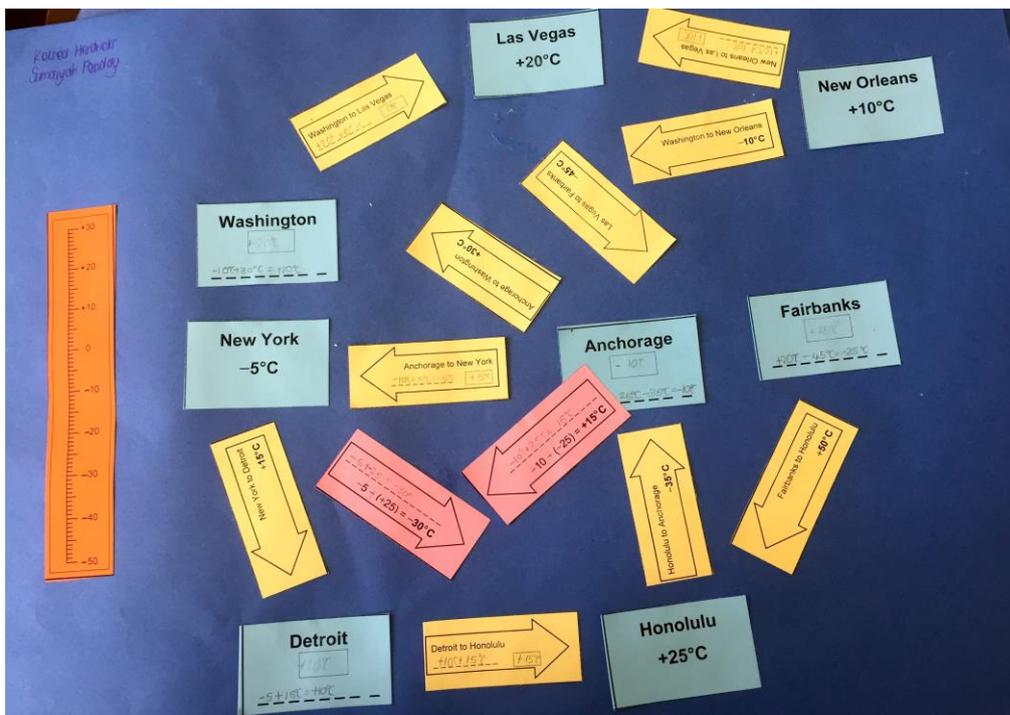


Figure 4: Cards glued onto a poster

Activity 5: Whole class discussion: going through the answers on the board.

Berenice called for attention again and invited the class to focus on the board. Berenice led the class in working out the solutions on each of the cards. Some learners were asked to come to the board to work out the solutions as well.

Activity 6: Whole class discussion

Berenice displayed a PowerPoint slide (Figure 27) with three more pairs of cities to compare and led the class in a discussion. Learners were then given two questions to work on as homework. The questions were similar to the calculations the learners were working on during the lesson and they involved filling in gaps. Berenice specifically asked the learners to give explanations of how they decide to calculate the sums. Berenice also asked the learners to check if their answers make sense.

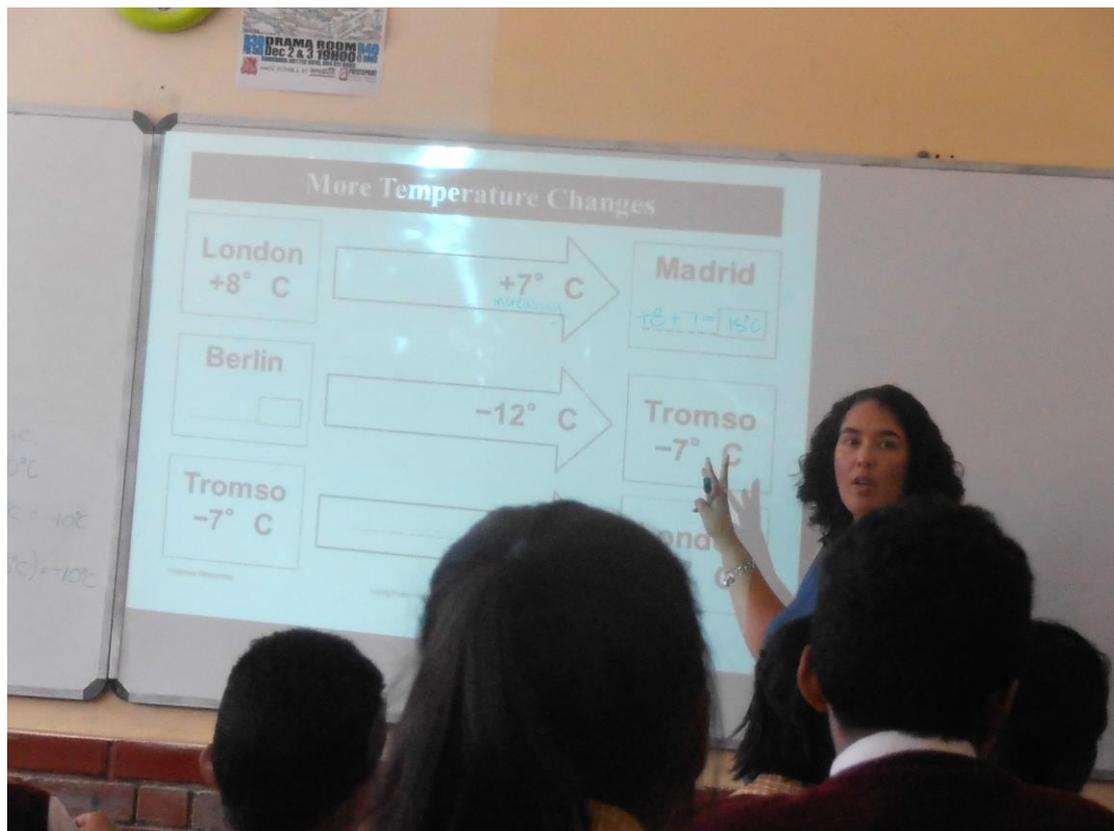


Figure 5: PowerPoint slide of cities to compare

Lesson 4: 11th March 2015 (not observed)

Activity 7: Students completed a questionnaire about the lesson. The results are reported in Section 5 – Pupil perceptions.

Research lesson 2 (Interpreting equations)

Lesson 1: 24th May 2015

Activity 1: The students were given the questions shown in Figure 13 (see above) to answer individually. Berenice took in the worksheets and assessed them. She did not give the assessed worksheets back but at the beginning of the next lesson she gave the learners verbal feedback. She said that some learners had used the incorrect variables.

Lesson 2: 25th May 2015

Activity 2: Class discussion: what's the right equation.

Berenice showed the PowerPoint from the lesson on the data projector (Figure 14 above). The first slide was $y = 4x$ and asked which variable is greater. One learner said that y is great than x . She asked the learner to explain why. He said, "if x is a number then y is 4 times bigger than that number". She confirmed that he was correct.

She showed a picture of the number of eggs (e) and the number of egg boxes (b) and asked the class to make an equation that links e and b (Figure 28).

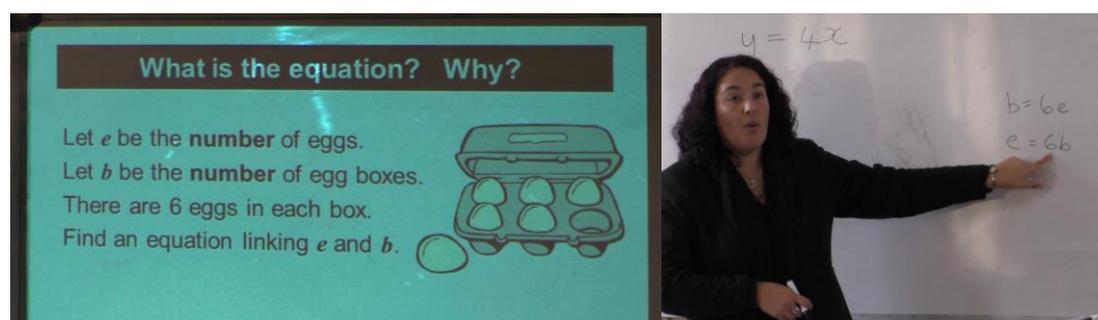


Figure 6: Which equation correctly links e and b ?

Berenice asked a particular learner (Ridwaan, real name) who said $b = 6e$. Another learner said $e = 6b$ (Figure 28). Berenice suggested that they use specific numbers to test which of the two equations was correct. She said 'in the picture there are 6 eggs and 1 box.'

One of the learners suggested the top equation ($b = 6e$) could be changed to $b = e/6$. Everyone appeared to agree.

Berenice then checked the second equation. She asked the class to give her an example to show that the second equation works. None of the learners answers so she used the numbers from the picture (6 eggs and 1 box) to demonstrate that they satisfied the equation.

10:05 Another slide (figure 29) was shown on the data projector. Berenice pointed out that this time the **cost** of things was being considered. She asked 'if a can costs R10 how will you work out how much 3 cans cost?'. One of the learners said that you would multiply the number of cans by 10.

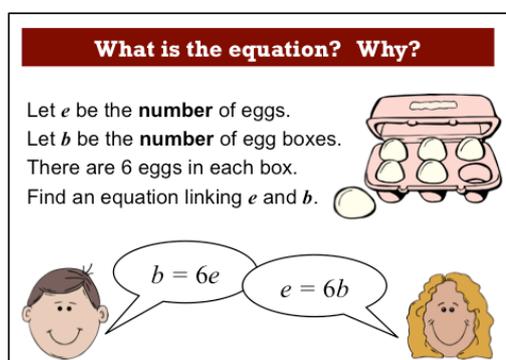


Figure 7: Second PowerPoint slide

Berenice explained that for this example they had to make an equation to explain the relationship between the **cost** of the eggs and the **cost** of the box. She emphasised that the cost per egg was the same whether the egg was bought individually or in a box.

One of the learners suggested cost of the box = 6 times the cost of an egg ($b = 6e$) and Berenice confirmed that this was correct.

Activity 3: Working in pairs

The students were asked to work in groups of three to match the cards shown in Figures 15 and 16.

Berenice explained that she was going to give each pair a set of cards. She used the big cards to show the class that the orange cards had a description in English and that the yellow (or green) ones had an equation.

She said: "It may be that you have two equations that match with one description or two descriptions that match with one equation. When you have decided which cards to pair you will make your own mini poster. I'll

give you a tip: there are 9 groups that should be matched.”

The cards and poster paper were handed out and the learners started matching the cards.

After a few minutes Berenice interrupted the class and said that she would write on the board what the variables represent as we had forgotten to make small cards with the definitions (see Figure 30).

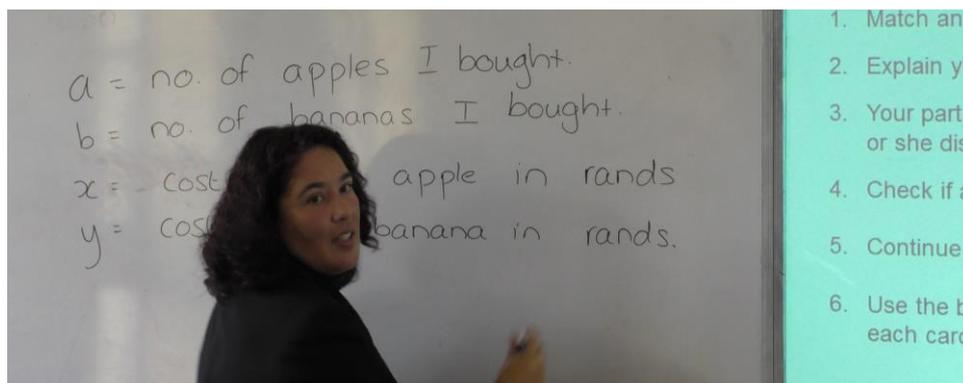


Figure 8: Berenice writing definition of variables on board

Berenice circulated and helped the groups to get started (figure 31). She asked a number of groups: “is there another way you can say the same thing?”

Some of the groups matched the cards very quickly although they were not correctly matched. Ingrid also circulated and told one group to check with the group in front of them.



Figure 9: Berenice questioning groups

Berenice told more than one group “I’ll give you another clue: there might be two pairs that go together. Two pairs that say the same thing in different ways.”

Activity 4: Institutionalisation

Berenice explained that she was going to hand out a recording sheet (figure 21, above) to record the matches that they had made.

Berenice stuck the big statement (orange) cards and the equation (green) ones on the board. She selected a particular orange card (*I bought 2 bananas*) and put it on the blank side of the board. A learner came up and chose a green card ($b = 2$) from the other side of the board where all the cards were stuck up.

The next card was “*A banana costs R2*” which was matched with $y = 2$.

“*Bananas cost twice as much as apples*” was first matched with $x = 2y$ (figure 32).

Berenice then asked the learner who had come up, ‘Which variable represents the cost of bananas?’ He said y . She then asked ‘Which cost more?’ to which he answered ‘bananas’. She then said ‘What is your equation saying if x represents the cost of an apple? It then says the cost of an *apple* is twice the cost of a *banana*.’ The learner didn’t respond and Berenice asked another learner to come and stick up their answer. A second learner chose $y = 2x$ (figure 32). The learner who had stuck up the incorrect answer originally then said ‘oh’.

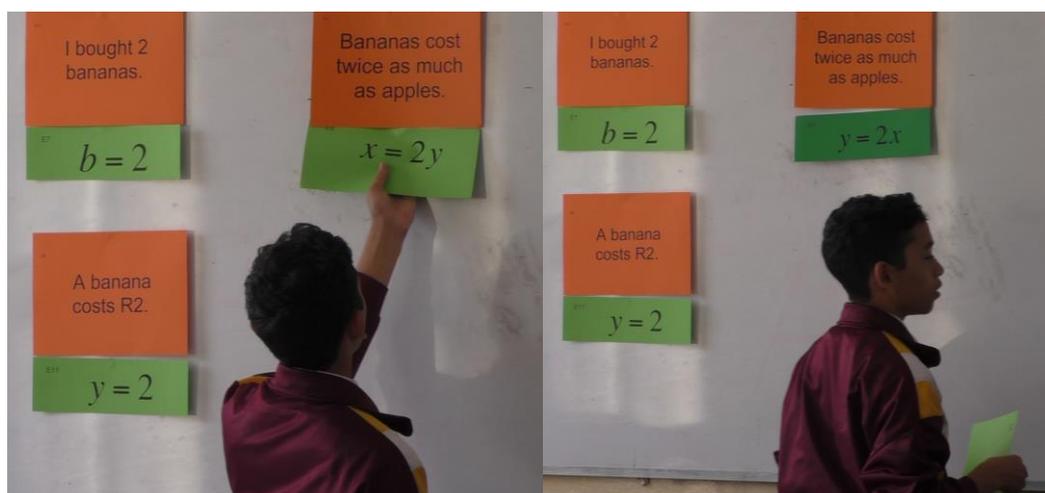


Figure 10: Learner replaces $x=2y$ with $y=2x$

A learner came to the board and matched “*Bananas cost half as much as apples*” with $y = \frac{1}{2} x$.

A different learner said she had another card that matched with it and she came to the board to match it with $x = 2y$ (figure 33).

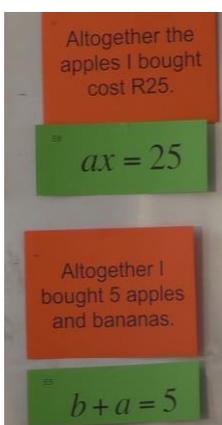


Figure 11: Two equations matched with 'Bananas cost half as much as apples'

After the lesson Berenice explained that she chose the first two cards because she thought the learners would find them easy. She said she chose the second two because they were similar and learners might have confused them (which they had).

Berenice then asked a particular group to choose an orange card and the matching green card. They chose 'Altogether the apples I bought cost R25'. The learner at board took a while before she chose $ax = 25$ to match it with (figure 34). Berenice told the learners in the class who were commenting that they should leave her to choose on her own.

After the learner had chosen, Berenice explained how to check whether it



was correct by asking 'how do we work out how much we spent on apples?'

'Altogether I bought 5 apples and bananas' was chosen by another group and matched with $b + a = 25$ (figure 34).

Some learners were talking to each other and appeared not to be concentrating on the work done on the board.

'Apples and bananas cost the same' was matched with $x = y$.

Berenice asked the class 'what in the equation tells you that they cost the same?' The learners answered that it was the equals sign.

Figure 12: Two more card matches

At this stage one of the learners said that there was a second equation that could be matched with ‘Bananas cost twice as much as apples’ and went to the board to add $x = \frac{1}{2}y$ (figure 35).

Another learner said that it should be matched with ‘Apples cost half as much as bananas’. Berenice pointed out that they were the same and that “if you flip the one you get the other. ‘Apples cost half as much as bananas’ is the same as ‘bananas cost twice as much as apples’”.

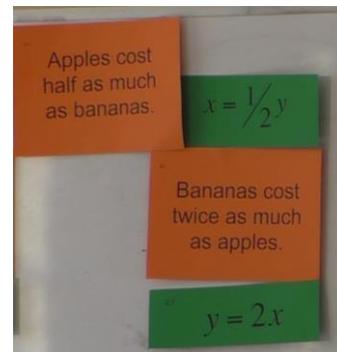


Figure 13: Four equivalent cards

A different learner matched ‘I paid R25 for all the apples and bananas I bought’ with $ax + by = 25$. The last learner matched the remaining cards: ‘I bought twice as many bananas as apples’ with $b = 2a$ and $a = \frac{1}{2}b$. Figure 36 shows the final completed matches.

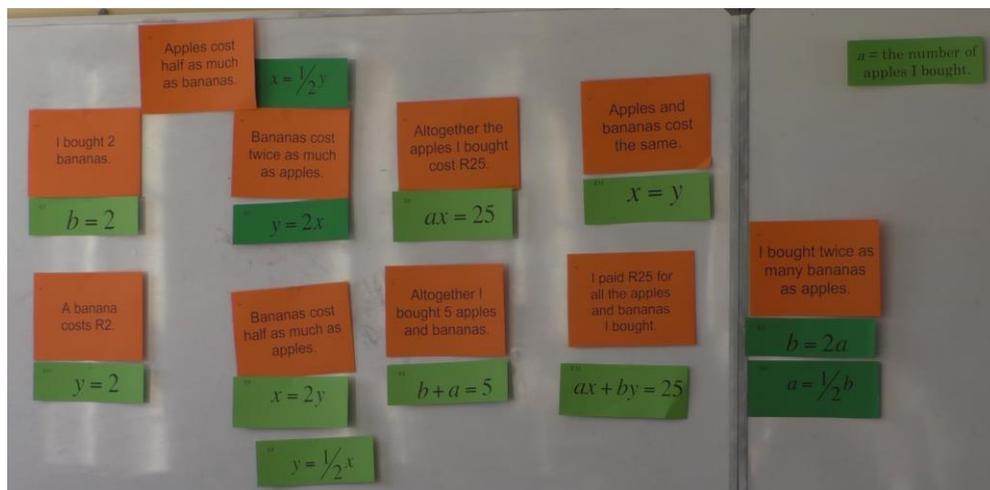


Figure 14: Complete card match on board

Berenice asked whether any groups had questions. No one had questions. The learners stuck their completed record sheets in their maths books.

Lesson 3: 26th May 2015

Activity 5: Students completed a questionnaire about the lesson. The results are reported in Section 5 – Pupil perceptions.

Research lesson 3 (Properties of Quadrilaterals)

Lesson 1: 24th August 2015

Activity 1: Introduction

Berenice began by telling the class that to revise the lesson they had done on Friday she wanted them to give her some properties of squares. The properties given by volunteers from the class can be seen in Figure 37.

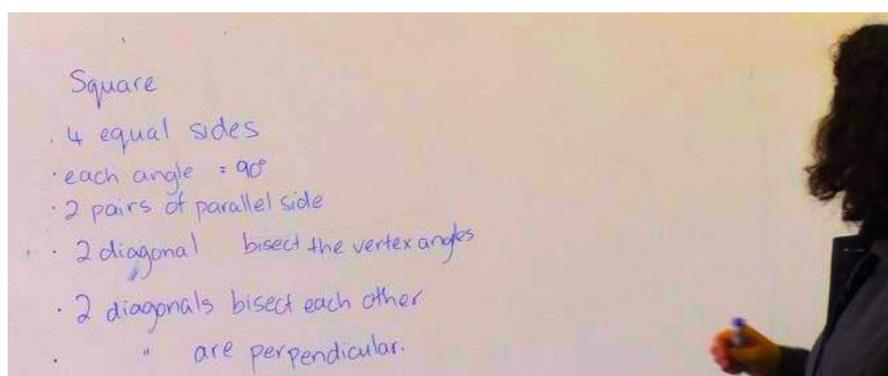


Figure 15: Properties of squares suggested by the class

One of the learners suggested 'more than one bisector'. Berenice asked the learner to explain what they meant and a different learner said 'more than one diagonal'. When a learner suggested 'diagonals are perpendicular', Berenice asked what this meant and then confirmed that it meant that the diagonals meet at 90 degrees.

Berenice asked the class to point out the properties that the square shares with other shapes. One of the learners said it shares the properties with a rectangle. When asked which properties it shares with the rectangle the learner said 'from the second one down' (see Figure 37) but after some prompting from Berenice he started going through each property and stating whether it was shared with a rectangle or not. For some he was sure and for others not. Berenice asked 'how would we know [whether the property is true]?' and the class suggested measuring and using tracing paper.

After the lesson Berenice told me (Ingrid) that in the previous lesson (on Friday) she had shown them how to use tracing paper to compare angles (to establish whether they were the same size).

Berenice asked the class ‘which properties do you need to know to be sure it’s a square?’ One of the learners said ‘the first one’ and Berenice put a tick next to it. She then asked ‘Do I need another property to make sure it’s a square?’ and a different learner said ‘all angles are 90 degrees’ and Berenice asked the class to think about whether that makes sense.

Activity 2: Working in small groups

Berenice said that they were going to continue looking at the properties of the family of quadrilaterals. She then showed them how to divide a piece of A2 poster paper into 6 equal parts.

Berenice then explained that each group would get 6 strips with properties of different quadrilaterals (figure 22, above). She told them to take a strip, put it in one of the parts of the poster paper, to decide as a group which quadrilateral the properties describe and then to draw and name it.

Berenice asked one learner to hand out the poster paper and another one to hand out the strips of card. The learners folded the paper and started working. Berenice and I circulated and made sure the learners understood what to do.

As she circulated, Berenice asked questions like:

Is there anything else that has 4 equal angles?

Could it be anything else?

Have you decided what it describes?

She encouraged groups to look carefully at the descriptions, to read them as a group and to decide on the quadrilateral as a group. She also suggested to a number of groups that they take a piece of scrap paper and draw the shape they were discussing to see if it satisfied the properties they were investigating.

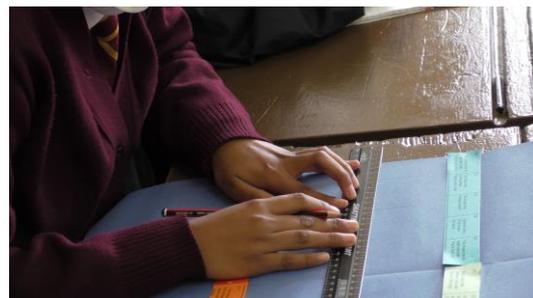


Figure 16: Learners working on their posters

Berenice helped a number of groups to use elimination to determine what quadrilateral was being described. She told them to 'look for properties that tell you it's definitely not this shape and it's definitely not that shape'.

With one group Berenice discussed the fact that equal and parallel are not the same thing. After the lesson she referred particularly to this misconception and said it was something that she would give attention to in the future.

She reminded a group of learners that they needed to be able to explain their decisions to the other members of their group.

She asked a number of groups which quadrilateral they thought was described by a particular strip. She then asked them why they thought so. For example, if the group said a strip described a parallelogram, she asked why it couldn't describe a kite or a rhombus.

She also asked some groups to show the properties of the quadrilaterals that they were drawing on their poster paper.

I (Ingrid) asked Berenice how she thought the lesson was going. She said 'I can see that some [learners] are pretty clued up and some aren't. They are getting to see why it should be one shape and not the other.' I suggested that the learners could be asked to fill in as many as possible of the properties of each quadrilateral.

Berenice said that the lesson was about to end and that they would continue with it the following day.

Lesson 2: 25th August 2015 (not observed)

Activity 3: Institutionalisation: Whole class discussion about correct answers.

Berenice reported that in the next lesson, she had handed out the posters to the students and had discussed the answers.

Activity 4: Students completed a questionnaire about the lesson. The results are reported in Section 5 – Pupil perceptions.

Analysis of the Research Lessons

Introductory comments

The lessons used by Berenice have already been analysed briefly above. The important point to make, at this stage, is that the design of the lessons lends itself to active learning. Further, the lessons are designed as formative assessment lessons, and although teachers and students may not use the information they gather in formative ways, it is likely that some formative assessment will take place.

This section provides an analysis of the lessons in terms of the teacher's approaches, the student engagement, the use of the technology and formative assessment.

The teacher's approaches

In this section we address questions such as:

- Did the teacher engage the students? Did the teacher promote student learning?
- How did the teacher give feedback to the students? (was the feedback inspiring for further inquiry or did the feedback limit further inquiry by providing solutions).
- How did the teacher motivate the students to become active learners in his/her classroom? What motivation tactics did the teacher employ to keep students focused on the subject?
- What type of questions did the teacher pose? And how did he pose them (orally, using technology, to the whole class, individually...)?

The teacher's role

The teacher's role before the classroom intervention was to work with the researchers to plan the lesson, deciding which parts of the proposed lesson to use and how. For example, she might decide to use a pre-lesson assessment or not. She might decide to cut short the whole class introduction to the main activity, to extend it or to adapt it in the light of the students' responses both before and during the lesson.

In the main lesson, the teacher's role was to introduce the main class activity, to supervise (in some way) the group work, and to bring the lesson to a close.

Pre-lesson assessment

For two of the lessons (first and second) the teacher gave a pre-lesson assessment task to the students, which aimed to find out something about the current levels of student understanding. In the first of these, Using Positive and Negative Numbers in Context, the assessment revealed that students found it difficult to explain in words how they knew the answer. Berenice used this information to plan her emphasis in the lesson. In the second, Interpreting Equations, she found that the students confused the variables, and it seems that this influenced how she introduced the task in the main lesson.

Introducing the activity

In introducing the activity, Berenice used the introductions suggested by the lesson guidance. Generally, she stood at the board and asked the class questions. Usually she waited for a show of hands before selecting a student to answer. Sometimes she selected more than one, and in this way she appeared to be successful in engaging the students and at the same time gathering information about what their current understandings were. For example, as described above, in the Positive and Negative Numbers in Context lesson, she held a discussion lasting about five minutes about the difference in temperature between two cities, asking three or four students to suggest an answer and, importantly, to explain it.

As an example of her questioning, in the Interpreting Equations lesson, when Berenice discussed the concepts needed for the lesson, she spent almost ten minutes, asking questions and apparently engaging the learners as they seemed eager to respond. We described above how she questioned the students, allowing them to offer different answer and not correcting them, just providing them with a strategy to establish whether or not they were correct.

It is probable that Berenice was using formative assessment, albeit informal and unplanned, during these discussions; through her questions she was gathering information about the students' understanding and reacting

accordingly. Berenice's formative assessment approach took the form of engineering an effective discussion and providing feedback that moved the learners on.

Main class activity

Berenice explained in some detail, in all lessons, how she wanted the students to work as a group, placing particular emphasis on the fact that they should take responsibility for one another's behaviour and understanding. For example, in circulating around the classroom, she encouraged students to explain their thinking to their partners and quite often, when she stopped the class to say something to the whole class, she said something similar to 'make sure your partner is listening'. Further, whereas many teachers we have observed using these lessons show little evidence of concern about the ways in which the students collaborate, Berenice did seem concerned. For example, as described above, in the first of the research lessons, she stopped the class and said that she had noticed that some pairs were placing the cards together but not taking turns. She pointed out that the instructions on the slide said that they should take turns and in this way was adopting the formative assessment strategy of activating students as resources for one another.

In all three lessons, the students were given sets of cards to work with. In all of them, the students seemed to be fully engaged by the activity: Berenice did not have any work to do in engaging them.

Berenice usually circulated observing what the students were doing. To begin with, her emphasis was on checking that they knew what to do and later she asked questions and gave feedback to move their thinking on. For example, in the second lesson (interpreting equations), as she walked around, she appeared to notice that many of the groups of students had not realised that some of the same colour cards were equivalent. She asked several groups, 'is there another way you can say the same thing?'. As another example, in the third lesson (quadrilaterals) she asked questions such as 'Is there anything else that has 4 equal angles? Could it be anything else?'. On the whole, she used these questions in a rhetorical way, not pausing long for answers, but seemingly suggesting how the students

should or could be thinking. Quite often she left a group saying ‘Think about it’.

Finishing off

In the final part of each of the lessons, Berenice went through the answers. We did not observe this part of the lesson for the third lesson. In the first lesson, she worked through the calculations with the students, asking for some students to come to the board and work out solutions.

In the second lesson, she used the big versions of the small cards. She had all the big cards stuck on the board, so that they could all be seen. She carefully chose the first two cards for the discussion, one because she thought the students would find it easy, and the other because she thought the students would find them confusing. She then went through the other cards. Throughout the discussion, she asked the students to choose the answers; she did not tell them. After each match, she intervened to comment, either asking if the learners agreed, or by asking a question such as ‘What in the equation tells you that they cost the same?’ In a way she was activating them as learning resources for one another and she was trying to make sure that they all remained engaged.

Student engagement

In this section we address questions such as:

- Did the students use the teacher as resource? Did the students use their peers as resource?

As stated, the lessons were designed to engage the students as they were required to match the cards, in their small groups. Our observations, and the reports of both the teacher and the students, suggest that the students were engaged. The student responses to the questionnaire in the second and third lessons highlighted phrases such as ‘made us discuss’ and ‘made me think’ which are, in our view, powerful indicators of engagement.

In terms of the students’ use of the teacher as a resource, we have already described above that the students did use the teacher as a resource, asking questions when they were uncertain of whether their matches were correct, for example.

Our main focus in the classroom was on the teacher so it was more difficult to see how students used their peers as a resource. However, we do have some data and it seems that peers were used as a resource within the small groups, between small groups and in whole class activity.

We observed that the misconceptions of one student was sometimes revealed by an incorrect matching of a card, and then the other student(s) would explain why it was wrong. More often, however, a student would choose a card, read it out and then suggest where it should be placed, explaining his or her reasoning.

In the whole class discussions, using peers as a resource was most evident when the students placed the cards on the board and there was some discussion about which cards were placed together. We did not observe the interactions closely but we did notice that there was some discussion and again it seems likely that it was about the mathematics. Hence we could claim that in some way peers were being used as a resource.

The use of technology

In this section we address questions such as:

- How did the teacher/students use the digital resources/technology?

As discussed, the technology we used was mainly card (small cards for the students to match and big cards for the teacher to use on the board). We also used mini whiteboards and Berenice used projection technology in some of the lessons.

In terms of *how* the technology was used, the small cards were arranged or matched according the requirements of the task. They were later stuck down onto a poster. The use of small cards provides information which the teacher or peers could use. We asked Berenice on a number of occasions how the cards had helped her and the students. She said that the design of the activity allowed her to observe what the students were doing and their placing, and moving, cards gave her an insight into their thinking.

The big cards helped support the whole class discussions at the end of the activities. Their role is as a sharing tool.

Formative assessment

In this section we address questions such as:

- Which teaching strategies, materials, techniques, ... were supportive (or hindering) in terms of the implementation/use of FA and ICT, and with respect to low achievers?

As discussed above, in the first two lessons, Berenice used the pre-lesson assessments and this provided her with information about the students' current levels of understanding.

In the lessons, there are many examples of strategies used by Berenice that support the implementation of formative assessment using technology. The use of big cards to model the activity and clarify success criteria supports formative assessment. The use of small cards provides information which can be used in the formative assessment of engineering effective discussions because the teacher gains some insight into the students' thinking, as described by Berenice. Similarly the use of small cards helps the teacher know what sort of feedback to give to move the learner on. The card activity demands, in a way, the formative assessment strategies of students acting as instructional resources for one another and students taking ownership of their own learning.

Appraising the lesson(s)

- How do you appraise the lesson? (in terms of FA and digital tools).
And how does the teacher appraise his/her lesson?

These lessons were carefully designed to provide opportunities for formative assessment, if the teacher does as the guidance suggests. As Berenice followed the lesson guidance quite closely for the most part, she was using formative assessment throughout, although she may or may not have realised that some of the strategies suggested by the guidance are aspects of formative assessment (e.g. group work means activating students as instructional resources for one another). It seems that Berenice took advantage of the opportunities for formative assessment in a natural way of 'good teaching' and, as she told us, she had come, through teaching these lessons, to appreciate the value of knowing what levels of understanding her students had.

In terms of the big and small cards (the technology used by us for formative assessment), both the researchers and the teacher thought they worked well. As Berenice said, the use of small cards allowed her to follow her students' thinking. The big cards were used in clarifying the criteria for success, or, as Berenice said '[The] big cards helped to prepare the learners for the activity and ensured that they understood instructions'.

5- Pupil perceptions

Q-sort

On 20 November Ingrid visited South Peninsula Secondary School to show a video montage to the two class we have been working with – Ms Jardine's class and Mr Ederies' class. The students were busy writing exams and so the only time available was on a Friday afternoon between 12:50 and 13:40 when there was an extended break while some of the students attended Mosque. This meant that there wasn't full attendance for the showing of the video.

After we showed the video most students were very keen to leave (as we were taking up their break time) but I asked for some volunteers from both classes to remain and give feedback about taking part in the research. Three students from each of the two classes remained behind. I handed out a piece of poster paper to each student, asked them to divide the paper into 4 columns and to label the columns 'strongly agree', 'agree', 'disagree' and 'strongly disagree'. I handed out a set of 44 statements to each students and explained that they were to place each statement in the column that best described their opinion of the statement. Unfortunately I forgot to bring Prestik with so Ms Jardine had to help me by looking for Pritt. In the end only one of the students didn't have Pritt. Unfortunately this resulted in her statement cards being knocked off her poster and onto the floor when she had already placed most of her cards and she had to redo the whole process.

Because all six students were working at the same time it was not possible to ask students why they decided to place cards in particular columns. What was interesting to note was that the three students from the one class spoke to each other throughout the whole activity. One of them would read out a

statement and then tell the others what they thought of the statement. It didn't appear that they were influencing each other – in other words, the other two did not then look for the same statement and put it in the same column – instead they seemed to want to share the process with each other. Interestingly, all three of these students agreed or strongly agreed with the statement 'I understand better if I work with friends in maths' while the three students from other class all strongly disagreed with this statement. As the students finished sticking down the statements they left. Ms Jardine and I collected the poster papers.

Questionnaires

For two of the research lessons, we gave the students a questionnaire about the lesson. The analysis of these is given below.

Research lesson 2

Berenice gave the learners in her class the questionnaire we had provided. She gave us 33 completed questionnaires when we went back to school to collect them.

The word cloud given in Figure 39, below, summarises the responses to the first question, which asked learners to choose three words/phrases that stood out for them in relation to the FaSMEd lesson.

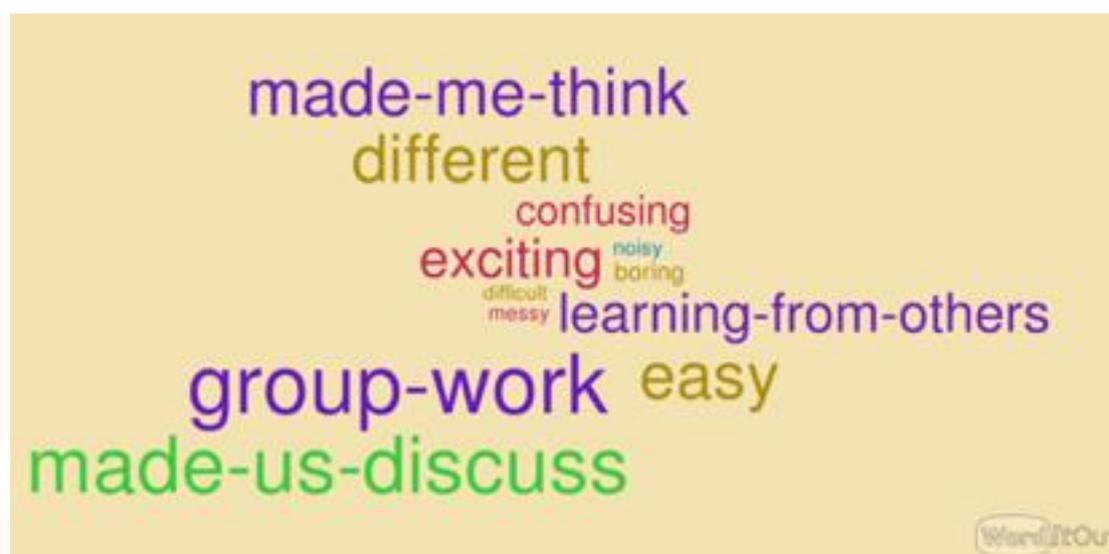


Figure 17: Words chosen by students about the FaSMEd lesson

This suggests that group work, to which 'made us discuss' and 'learning from others' are likely to be associated, were highly important to this class

(39 choices in all). It also seems that, although 'easy' was chosen by 12 learners, the activity still appeared to provoke thinking (12 responses) although of those that chose easy, only one also chose 'made me think'. Six of those who chose 'easy' also commented on the value of working in groups. Twelve learners chose 'different' with nine choosing 'exciting'.

The learners were asked to complete the sentence 'The card matching activity was different because ...'. Three simply wrote the words they had chosen for the first question. Of the remainder, eight remarked that this sort of lesson is unusual, often suggesting that the 'never do this' (for example). One said 'we usually just write in our books' and another explained 'We never do this kind of work. It really helped me because I never understood the work before this lesson'.

Twelve learners made comments about working in groups, with two of these saying that they got to interact with others. Some explained what they liked about working in a group. One, for example, said that 'It helped by showing how others think' and another stated that 'we could work together and learn from each other'.

Four learners, made comments about how the activity made them think, although they had all chosen 'made me think' in the first question. However, it seemed they wanted to explain further. One, for example, said 'it made everyone think and made the lesson exciting' and another said 'it was work that made you think and used your hands to paste things' (direct quotes).

Two learners referred to the teaching methodology, with one stating that they had used cards and the other saying 'we got to put cards on the board'.

In terms of the mathematical task, one learner said that it was like a 'puzzle', another explained that 'some equations had more than one answer and some equations were the same just in different words'. One learner remarked 'we had to match the proper equation to the sentence and there were 4 variables, 2 pertaining to cost and the other 2 pertaining to amount'.

A final two comments related to the fact that the lesson was being video recorded.

The learners were also asked to complete the sentence 'The card matching lesson would be better if ...'. One response was blank. Nine referred to working in groups, with six of these saying it would be better if the groups

were bigger and two saying it would be better if they could work individually. The last one said ‘... if I wasn’t paired up with the most annoying person in the class’.

Four responses suggested that technology could be used, but explained no further.

Five had suggestions about the teaching approach, with one saying that it would be better ‘if we did not paste the cards’, one saying ‘if we compared other work to our own, learning from others’ (underlined in the original), one suggesting more discussion and one saying that he or she would like to get the marks back and re-do if they had not understood. The last one suggested ‘if there was a prize for the first person done’.

Two learners suggested it would be better if it were easier, and another one suggested ‘if we didn’t have to think’. On the other hand, two learners suggested that they would like it to be more challenging. A further four, although not stating that it should be more challenging, suggested that there should be more cards and more equations. Another learner suggested ‘if all equations had completely different answers and two ways of doing it’.

The final five comments related to the learners practical experience of the classroom, with one saying ‘the children could be less noisy’, one saying ‘it wasn’t as messy as it was’, one suggesting it should be more organised, one suggested they should not be filmed and the final once saying ‘if it provided snacks’.

Research lesson 3

As before, we asked the learners to complete a questionnaire. We received 34 completed questionnaires. The first question asked them to choose three words or phrases that stood out most for them about the lesson. Figure 40 below provides a graphical representation of their responses, where the size of the word represents the number of times it was chosen.

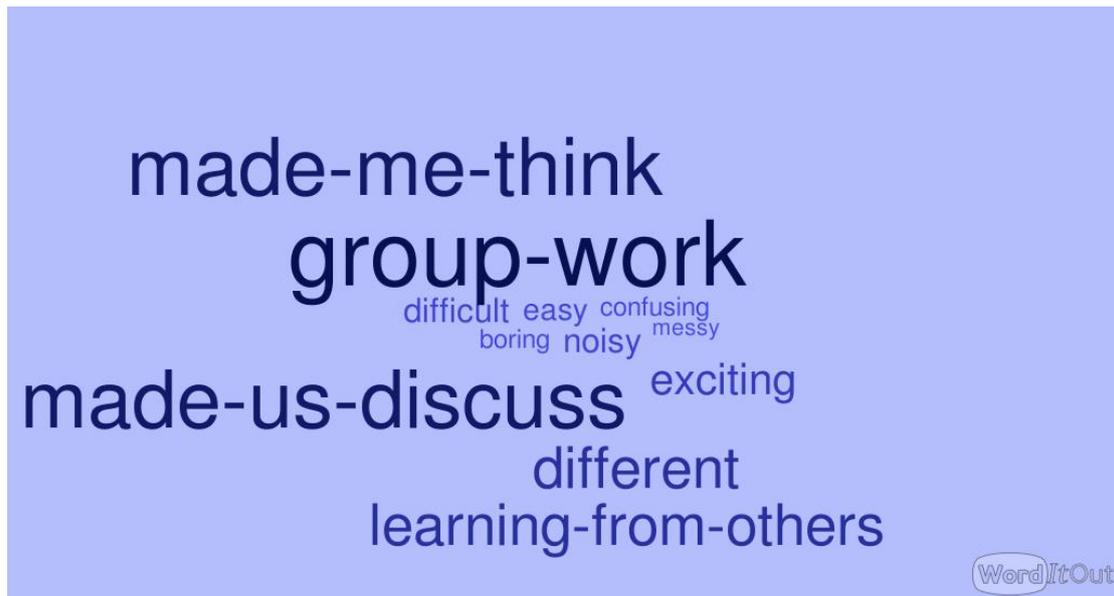


Figure 18: Frequency of words chosen by students

As can be seen, the most popular choices were ‘Group work’ (21), ‘Made us discuss’ (18) and ‘Made me think’ (18), followed by ‘Learning from others’ (11) and ‘Different’ (11). Of course, working in groups, discussing and learning from others are closely linked, and relate to the organisation of the lesson. Interestingly, of the 15 who did not choose ‘Group work’, eight mentioned group work in the following question, which asked them why the Fasted lesson was different, which means that only seven did not mention group work at all. Other responses to this question, related to group working, included eight that mentioned learning from their friends, needed to agree with one another and discussing the answers. In the third question, where they were asked how the lesson could be better, ten again mentioned group work, with six stating that bigger groups would be better, three stating that pair work would be better and one saying that individual working would be better. Two mentioned the need for partners who were in some way ‘better’. Clearly, for this class, working in groups was important, whether or not they liked it.

In terms of the activity itself, ‘Made me think’ was chosen by about half the respondents. Nine referred to the activity in the second question, with four saying in various ways that, for example, it made their ‘brains work’. Two referred to the active/practical nature of the task and two stated that it was fun. One, for example, said that it was ‘a little mind challenging’ and another said ‘we never had to think this hard before’. On the other hand, two

mentioned that the lesson would have been better if the task was more challenging.

Other comments related to how the activity would be better included lack of time (three comments), and the possible use of ‘electronics’ and IT (three comments). Other learners mentioned having a competition (1), less messy (1), less confusing (1), more help from adults (2), the chance to compare answers with other groups (2) and marking the work (1).

Overall, it appears that the learners enjoyed the lesson, and certainly our observations and the teacher’s comments suggest that they were very engaged. It is less easy to be confident about how much learning took place.

Conclusion

Our final remarks aim to sum up some responses to the research questions:

- How do teachers process formative assessment data from students using a range of technologies?’
- How do teachers inform their future teaching using such data?
- How is formative assessment data used by students to inform their learning trajectories?
- When technology is positioned as a learning tool rather than a data logger for the teacher, what issues does this pose for the teacher in terms of their being able become more informed about student understanding?

Berenice, in teaching three FaSMEd lessons, mainly used technology in the form of small and big cards for formative assessment. The technology provided support in clarifying the learning intentions and criteria for success, as well as whole class and small group discussions, engineered by Berenice as part of her formative assessment. It is difficult to say *how* Berenice processed the data the technology provided. However, we can say that much of this processing took place in real-time in the course of the lesson(s), and almost certainly will have influenced the focus of her questions, feedback and discussion with small groups and the whole class.

Where ‘future’ teaching is mentioned, we have already discussed future teaching in terms of the immediate future; within the same lesson. In terms

of future teaching with this class, there is some evidence that data collected during the research lessons might have influenced future lessons as Berenice did, on occasions, mention that she would pay more attention to specific misconceptions in a later lesson. In terms of teaching the same lesson another time, to a different class, we suggest that formative assessment data (i.e. how the students responded to the activity) would be one of many factors, including the lesson design, that might influence his decisions.

For the students, it seems unlikely that they would be interested in formative assessment data. It appeared that they were very engaged in the tasks we gave them, and they saw their role as completing the task (as they should, (Sierpinska, 2004)). We can say that we saw them using formative assessment from their peers to inform how they worked together with them, but cannot say anything about how they used formative assessment to inform their own learning trajectories.

The technology we used can be seen as a learning tool rather than a data logger. We have discussed how the use of small and big cards, particularly the small cards, gave the teacher valuable information about the students' understanding and we have explained how the teacher used this information.

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